



Philadelphia University
Faculty of Science
Department of Mathematics
Mid-Term Exam



Instructor:
Dr. Abdullah Alsoboh

S.N. :

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Semester: 1st Semester

Course Name: Real Analysis 1

Course Number: 250311

Date: 30/04/2024

Duration: 60 Min

Name:

I.D. Number:

Q₁: [/5 marks] Circle **True** or **False**. Read each statement carefully before answering.

1. True False. If x and y are real numbers with $x < y$, there exists an irrational number z : $x < z < y$.
2. True False. If $a > b$ and $c < 0$, then $ca > cb$.
3. True False. Every bounded sequence is convergent.
4. True False. The sum of two divergent sequences diverges.
5. True False. A monotone sequence of real numbers is divergent.

Q₂: [/5 marks] This question consists of 5 multiple choice questions (1 Mark for each), where each question has 4 options. Put the answer symbol in the table below.

Question #	1	2	3	4	5
Answer Symbol					

1. If every nonempty set of real numbers that has an upper bound also has a supremum in $\mathbb{R}$. This property is:
a) The Order Property of $\mathbb{R}$ b) The Completeness Property of $\mathbb{R}$
c) The supremum Property of $\mathbb{R}$ d) The Archimedean Property
2. If $x \in \mathbb{R}$, then there exists $n_x \in \mathbb{N}$ such that $x \leq n_x$. This property is:
a) The Order Property of $\mathbb{R}$ b) The Completeness Property of $\mathbb{R}$
c) The supremum Property of $\mathbb{R}$ d) The Archimedean Property
3. Which of the following is **NOT** a property of the supremum of a set of real numbers?
a) It is an upper bound of the set. b) It is the smallest upper bound of the set.
c) It may or may not belong to the set. d) It is unique.

4. $\inf \left\{ \frac{1}{n} - \frac{n}{n+1}, \quad n \in \mathbb{N} \right\} =$

a) 0

b) -1

c) 1

d) $\frac{1}{2}$

5. The sequence (a_n) is defined as $a_n = \sqrt{n+1} - \sqrt{n}$. What can be said about this sequence as n tends to infinity?

a) It converges to 1

b) It converges to 0

c) It converges to $\sqrt{2}$

d) It diverges to infinity

Q₃: [/6 marks] Consider the sequence $a_n = \frac{4n+1}{3n+1}$.

1. Write the first four terms of a_n .

2. Use the definition of the limit of a sequence to establish $\lim_{n \rightarrow \infty} \frac{4n+1}{3n+1} = \frac{4}{3}$.

Q₄: [/6 marks] Evaluate the following limits (Show the details of your work)

1. $\lim_{n \rightarrow \infty} \frac{2}{4^{n+1}}$

2. $\lim_{n \rightarrow \infty} \frac{\cos(3n - 1)}{4 - 2n}$

3. $\lim_{n \rightarrow \infty} \frac{\cos(2n)}{n^2 + 1}$

Q₅: [/4 marks] Let $x_1 = 4$ and $x_{n+1} = 1 + \frac{1}{2}x_n$.

1. Show that x_n is monotone and bounded.

2. Is x_n convergent? explain. If so find limit.

Q₆: [/4 marks] Prove that $\sqrt{3}$ is not a rational number.